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SLURRY FLOW METERING BY PRESSURE DIFFERENTIAL DEVICES

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Abstract—A simple system for measuring slurry concentrations and flow rates by pressure difference devices has been studied. Solids concentrations have been determined by pressure measurements in vertically upwards and downwards sections of the flow, and flow rates measured by a venturi meter located alternately in horizontal, vertically upwards and vertically downwards flows. The venturi performance has been described in terms of a discharge coefficient based on mixture density, and the concentration measurements have been compared to values determined by weigh tank samples. A range of sand slurries with median particle size from 150 to 740 μm and an ilmenite slurry with a median particle size of 170 μm have been tested. A limited number of tests on a 17 μm sand slurry have also been recorded.

For fine sand slurries (150 μm or below) it was found that the water value of venturi-discharge coefficient applied, so that the accuracy of flow measurements will be limited only by the accuracy of the pressure and specific weight measurements. Experimental values of specific weight from the vertical loop section were on average within 1% of the values from the weigh tank, where the majority of this scatter was caused by the limitations of the weigh tank method.

For coarser slurries, the venturi-discharge coefficients depended primarily on solids concentration, with a secondary dependence on Reynolds number, and values were below the water value. An analysis of the flow based on a one-dimensional momentum balance indicated that relative velocity between the phases could account for these low values of discharge coefficient. This was verified by high-speed photography of the flow which suggested a movement of particles towards the centre of the flow at the throat.

The overall indications are that the combination of a vertical loop section of flow (for concentration or specific weight measurements) and a venturi meter (for flow rates) provides a simple and accurate means of metering slurry flows. For fine slurries only water calibrations or calibrations from Standards are required.

INTRODUCTION

The hydraulic transport of solid materials in pipes is a well established technique, particularly in the mining industry. Long distance pipelines such as the 85 km long iron-ore pipeline at Savage River, Australia and the 430 km Black Mesa coal pipeline in the U.S.A. have proven the viability of this means of transport. Of equal importance, however, are the multitude of short-distance slurry pipelines found within processing plants for transporting solids between various treatment stages. In each of the above situations it is the solid material which is the product being transported, so that some means of determining the solids throughput, in say tonnes per hour, is essential. The best known and perhaps most widely accepted system for achieving this measurement is to combine a nuclear density gauge, which can be calibrated to give percent solids in the flowing slurry, with a magnetic flowmeter, to give the total volumetric flow of slurry. Peirce (1962) suggests that this type of system is "unexcelled in accuracy of slurry measurement and rangeability. It is well beyond the development stages, with systems having been in continuous operation for a number of years". Behrend (1973) more critically analysed the operation of this type of system when reporting on the results of a workshop on "Analysis and Control in the Canadian Minerals Industry" attended by groups from within the mining industry. He noted that the overall opinion of the workshop was that this type of system "is woefully inadequate", although many installations have been accepted as satisfactory. At best, monitoring of the slurry flow by this method is expensive when compared to single-phase flow techniques, and is only employed either on large pipeline installations, or on shorter pipelines where knowledge of the flow is critical. Other limitations include the facts that magnetic materials affect the calibration of the flowmeters, and nuclear density gauges are only accurate for homogeneous, or well mixed flows, which usually requires installation in a vertical section of flow. There are thus many applications for which these meters are unsuitable, or where the expense is unwarranted. With these factors in mind, several studies have been made of simpler and less expensive systems for measuring slurry flow rates and concentrations. Of particular interest here is the use of a venturi meter for measuring the flowrates in conjunction with a vertical loop section of pipe for determining solids concentration. The vertical loop method, often referred to as a counterflow meter, relies on measurements of static pressure drop over identical lengths of vertically upwards and vertically downwards flow. By defining the mean delivered solids concentration as the average of the in-line concentration in the upwards and downwards sections of flow, this value can be found by subtracting the pressure difference values. For water filled manometer lines, the difference is directly proportional to the solids concentration. Assessments of this method have been made by Brook (1962), Einstein & Graf (1966) and Weisman & Graf (1968) and a large scale installation (pipe diameter = 0.685 m) for determining concentrations in a dredging operation was reported by Van der Veen (1972). Brook experimented with two coarse slurries containing 9.5–12.5 mm bakelite particles and 6.5–9.5 mm basalt respectively. When comparing concentration estimates from the counterflow meter and a sampling tank, he found that estimates of solids concentration were within about 20%. Brook also showed that if an estimate can be made of frictional pressure gradients, the same pressure measurements can be used to give an estimate of the flow rate. Unfortunately, the estimated flow rates by this method showed large errors when compared to values from weigh tank samples, especially for the low velocities (below about 2 m/s). He suggested that this discrepancy resulted from the neglect of relative particle to fluid velocities in the analysis. Einstein & Graf (1966) also experimented with a vertical counterflow meter for two sand slurries (median particle size = 1.15 and 1.7 mm), and in their analysis accounted for relative velocities effects. In a later discussion on these results Weisman & Graf (1968) reported that the maximum error between calculated (from pressure measurements) and measured concentrations (from weight samples over small time intervals) is 5%, while inaccuracies in flow rate measurement were of the same order of magnitude as those reported by Brook (1962).

The dredging operation referred to above is a large reclamation project near Amsterdam, and Van der Veen (1972) reported on the successful operation of the vertical loop system after half a year of operation. The vertical loop section of pipe also incorporated two magnetic flowmeters to allow for estimates to be made of the total solids throughput and over a 6 month period the total throughput estimated by this measuring system was 3.72% below the estimates according to "conventional" means (volume of barges as well as echosounding in the deposit).

These various results suggest that the vertical counterflow meter is capable of estimating solids concentrations, but because of the limited amount of data available the accuracy has not been fully established.

Several experimental studies have been previously reported for slurry flow metering with venturis, but once again only limited data are available. Brook (1962) investigated the use of venturi meters for measuring flow rates with the same slurries referred to above (for testing the vertical counterflow meter). With venturis mounted both vertically and horizontally, Brook found that flow rates through the venturi could be estimated, provided the slurry density (and hence solids concentration) was known, and provided a value of the venturi discharge coefficient was known. Calibration of the venturis gave discharge coefficients generally above the equivalent water discharge coefficients for horizontal flows and values scattered around the water values for vertical flow. Graf (1967) experimented with sand slurries (median particle size = 1.15 and 1.70 mm) and suggested that total pressure loss measurements, as well as conventional inlet to throat measurements used for flow rate determination, provide additional information on the flow. He used the total head loss to estimate the solids concentration, concluding that the venturimeter can be used for determining both slurry flow rate and solids concentration. Discharge coefficients which can be calculated from Graf's tabulated results are significantly greater than water values for both of the slurries tested. Shook & Masliyah (1974) presented a one-dimensional analysis of flow through a venturi, demonstrating that the relatively high discharge coefficients reported above could be accounted for by the fact that particles entering the venturi throat do not accelerate as quickly as fluid entering the throat, due to their greater density. With a limited amount of experimental data on silica sand and a lead shot slurry they qualitatively verified the analysis by noting that relatively high discharge coefficients applied for the lead shot slurry. For the sand slurry ($d_{50} = 430\,\mu$m) the discharge coefficients were not significantly different to water values for either vertical or horizontal flow except for low velocity results (below about 2 m/s), and if any difference could be suggested for the vertical flow situation it is that slurry discharge coefficients were below the water values.

As a result of these limited number of experimental studies with the vertical counterflow meter and venturi meters, it would be reasonable to suggest that a system incorporating these measuring techniques could be adopted for some slurry flow applications after careful calibration. However the range of applicability has not been determined, nor has the degree of accuracy which can be expected. This paper presents the results of a study aimed at resolving these uncertainties by comparing concentration estimates for a range of slurries with weigh tank values and by comparing venturi discharge coefficients for slurries with water values.

PRINCIPLES OF MEASURING DEVICES

Venturi meters have been used and calibrated extensively for measuring single-phase flow rates, and the principles of operation and necessary calibrations are well documented in British Standard 1042: Part 1: 1964. The venturi device examined in this study is illustrated in figure 1, for which the flow rate of a homogeneous mixture flowing horizontally is determined by an equation of the form

$$Q = C_D A_2 E \sqrt{\left(\frac{2(p_1 - p_2)}{\rho_m}\right)}, \qquad [1]$$

where A = area;
C_D = discharge coefficient;
d = diameter;
$E = (1 - m^2)^{-(1/2)}$;
$m = d_2^2/d_1^2$;
p = static pressure;
Q = volume flowrate;
ρ_m = mixture density.
Subscript 1 = inlet;
2 = throat.

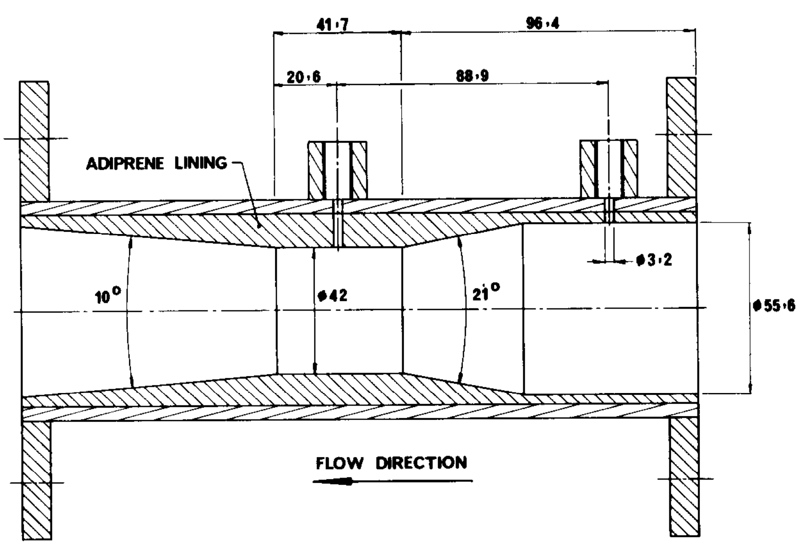

Figure 1. Details of venturi (all dimensions in mm).

The discharge coefficient is basically a constant with a slight dependence on area ratio, inlet diameter, and Reynolds number, as described in the standards.

The vertical loop, or counterflow meter, consists of a vertically upwards and vertically downwards section of pipe, as illustrated in the schematic diagram of figure 2. With subscripts denoting the location of pressure taps as defined in figure 2, the pressure difference over each length of pipe (h_u and h_d) for water filled tapping lines may be written as:

$$h_u = (p_3 - p_4)/\gamma - L = h_f + Lq_u(s-1), \qquad [2]$$

and

$$h_d = (p_5 - p_6)/\gamma + L = h_f - Lq_d(s-1), \qquad [3]$$

where h = head loss in each length of pipe (metres water);
q = in-line volume concentration of solids;
s = specific gravity of solids;
L = distance between tapping points (metres);
γ = specific weight of water.
Subscripts f = friction component;
u = upwards section;
d = downwards section.

By assuming that the delivered concentration C is equal to the mean of the in-line concentrations in the upwards and downwards portions of flow, then

$$C = \frac{h_u - h_d}{2L(s-1)}, \qquad [4]$$

which comes from [2] and [3]. The only other assumption required is that the frictional head losses in the two portions of pipe are equal, and the validity of this will be reflected by the accuracy of the experimental values of concentration.

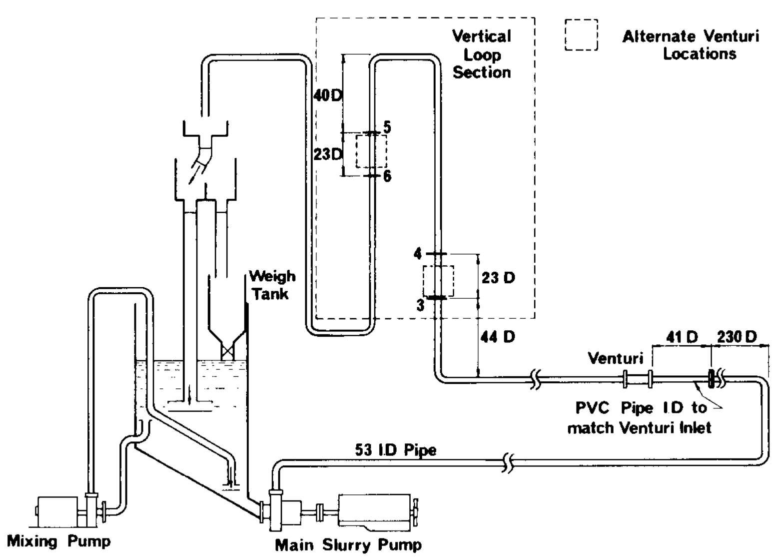

Figure 2. Location of venturi and vertical loop section in slurry. Test facility.

EXPERIMENTAL

The slurry loop system is shown schematically in figure 2, consisting basically of a 50 mm diameter pipe with a centrifugal pump driven by an industrial petrol engine for speed control. Volume flow rates and delivered solids concentration were measured by sampling a portion of the discharge in the weigh tank. Altogether, five sand slurries, $s = 2.65$ and an ilmenite slurry, $s = 4.47$ (see figure 3 for size distribution) were tested and the delivered solids concentration calculated by [4]. The flow measurements with the weigh tank were used to determine the venturi-discharge coefficient from [1] for each of the slurries with the venturi located alternately in the horizontal, vertically upwards and vertically downwards sections of flow. Pressure differences were measured with a KDG series 400 differential pressure transducer/transmitter.

The allowable sample period for each particular slurry was a matter of compromise. The larger the sample, the more accurate the volume and weight measurement, and hence the more accurate was the concentration estimate. Also switching and timing errors in the electronic timer would be minimised. However, because of the settling nature of most of the slurries tested, the mixture in the main tank was not homogeneous and diversion of the discharge to the weigh tank altered the concentration of the mixture entering the pump. For excessively long sample periods, this lead to a reduction in the pump load and hence an increase in speed and flow rate. For the coarsest slurry (sand E of figure 3) sample periods were maintained at around 10 s, while periods as high as 30 s were acceptable for the finer slurries. The effect of the increase in pump speed was to indicate high values of discharge coefficient.

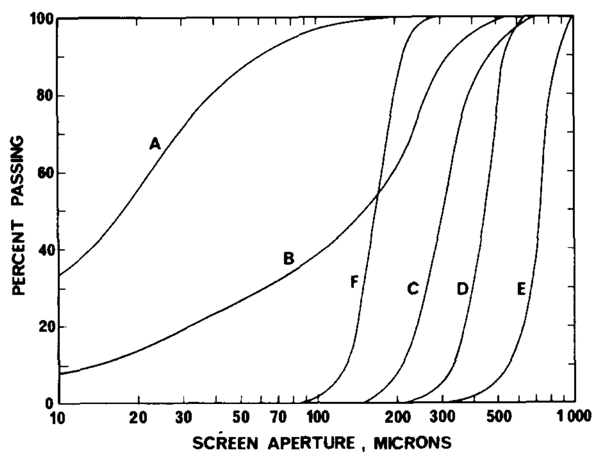

Figure 3. Screen analysis of slurries.

	Material	Median size (μm)
A	Fine sand	17
B	Ground sand	150
C	River sand	310
D	25–52 mesh sand	450
E	16–30 mesh sand	740
F	Ilmenite	170

EXPERIMENTAL RESULTS

Before and during the experiments with the slurries, the venturi meter and the vertical loop were calibrated with water. During the entire test programme and venturi-discharge coefficient for water did not change and was equal to 0.987 with a standard deviation of 0.003 for values of Reynolds number above 2×10^5. Below this value of Reynolds number, discharge coefficients were lower. To check the repeatability of the overall flow measuring system, 22 measurements of discharge coefficient were made approximately half-way through the slurry test programme for varying water temperatures and nominally the same Reynolds number, giving a discharge coefficient of 0.9868 with a standard deviation of 0.0028. This repeatability of results was only obtained because of the wear resistance of the Adiprene (a polyether urethane) lining. Water tests of pressure drop over the two vertical lengths of pipe showed a discrepancy which depended on flow rate. This resulted from the fact that the pressure tapping holes were drilled into commercial steel pipes which have relatively rough inside surfaces. Initial errors appeared quite large but after the tappings were re-drilled and all burrs were removed, the discrepancy was approx. 0.028 times the velocity head. No compensation was made for this error in the concentration measurements.

The discharge coefficients for the three orientations of the venturi are presented in figures 4–6. For the venturi mounted horizontally the discharge coefficients appeared to be equal to the water value until a critical concentration, at which the discharge coefficients began to decrease with increasing concentration. This critical value of concentration was higher for the finer slurries. The results suggest that for sand slurries with finer particles than sand C ($d_{50} = 310\,\mu m$) the water discharge coefficient is applicable for concentrations up to at least 30% by volume (53% by weight). The ilmenite slurry also gave values of discharge coefficient grouped around the water value and this is not unexpected since the settling velocity of a median sized ($d_{50} = 170\,\mu m$) ilmenite particle, 0.036 m/s is slightly below the median sized particle of sand (0.045 m/s), suggesting that the ilmenite slurry will behave in a slightly more homogeneous manner.

With the venturi mounted vertically, discharge coefficients were generally below the water

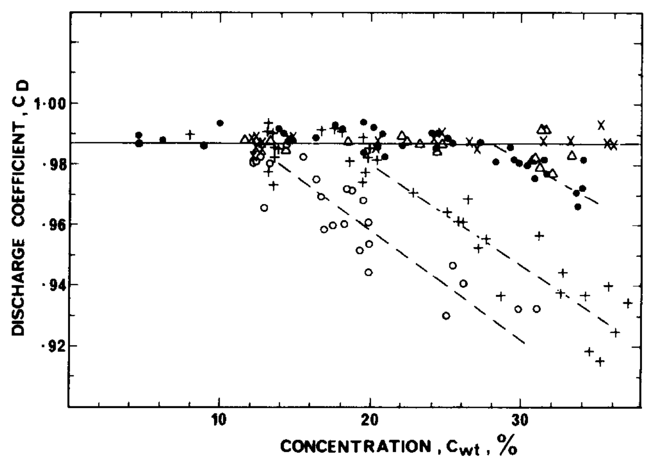

Figure 4. Venturi discharge coefficients—horizontal flow. ∇, slurry A; ×, B; ●, C; +, D; ○, E; △, F.

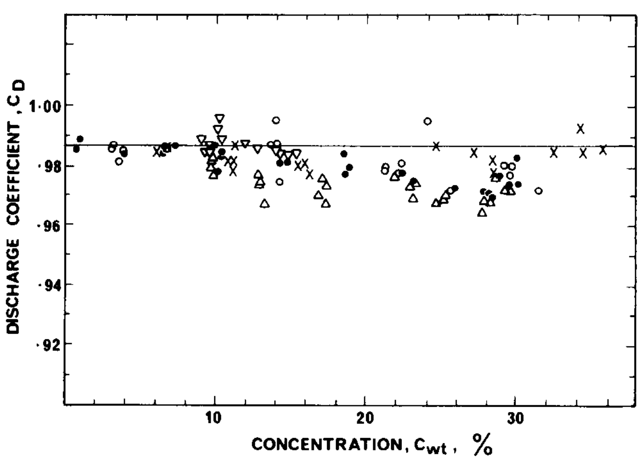

Figure 5. Venturi-discharge coefficients—vertically upwards flow. Symbols as in figure 4.

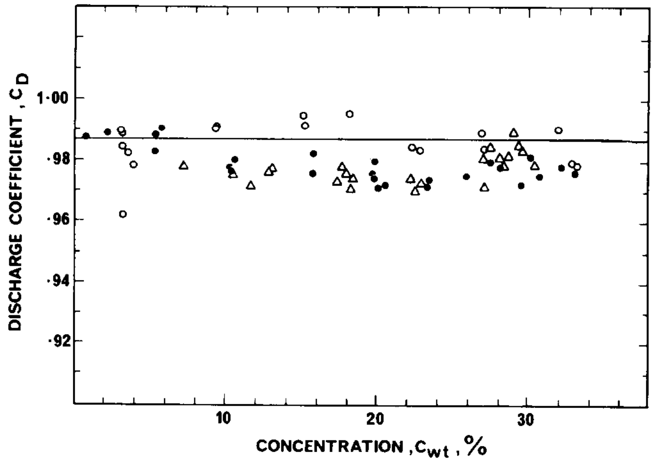

Figure 6. Venturi discharge coefficients—vertically downwards flow. Symbols as in figure 4.

values. In both upwards and downwards flow, the ilmenite values were similar to the values for sand C, with values up to 1.5% below the water value. The discharge coefficients for sand C, with values up to 1.5% below the water value. The discharge coefficients for sand E were grouped around the water value which will be discussed later.

Although the discharge coefficients have been presented as a function of concentration, there is also a secondary dependence on flow rate, and hence Reynolds number. The magnitude of the variations with Reynolds number can be gauged from figure 7 which shows the variations of discharge coefficients for sand D at different concentrations. The Reynolds number here is based on the mixture velocity and water viscosity and density. It does appear that there is an increasing dependence on Reynolds number for higher concentrations where the discharge coefficients are furthest from the water value. For sand B, which gave discharge coefficients equal to the water value all values of discharge coefficient were within 0.5% of the water value for horizontal flow, and showed no Reynolds number dependence.

The estimates of solids volume concentration by the weigh tank (C_{wt}) and the vertical loop (C_{loop}) were found to be in general agreement for all slurries. The results are summarised in figure 8 which shows values of the ratio C_{loop}/C_{wt} as a function of C_{wt}. There is general agreement between the values with the loop values tending to be marginally below the weigh tank estimates. Only one pair of estimates showed a deviation greater than 10% and the mean absolute difference was 3.2%. In determining the accuracy of the device, however, it is more realistic to compare estimates of specific weight since this is the quantity being measured. The mean absolute difference of the specific weight estimates was 0.93% for the 81 values recorded. These

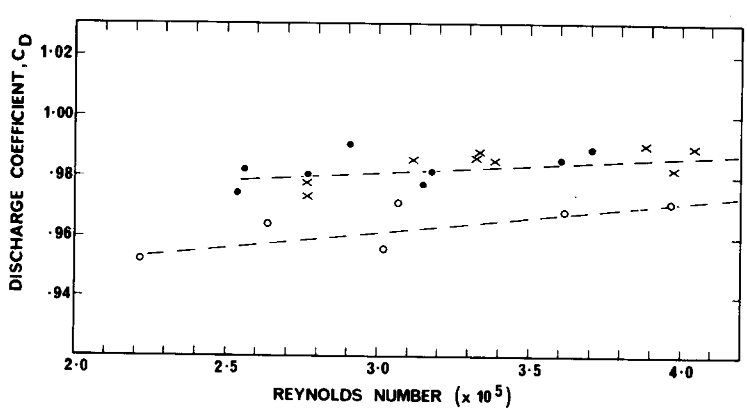

Figure 7. Variation of discharge coefficient with Reynolds number for slurry D. ×, 12% concentration; ●, 20%; O, 25%.

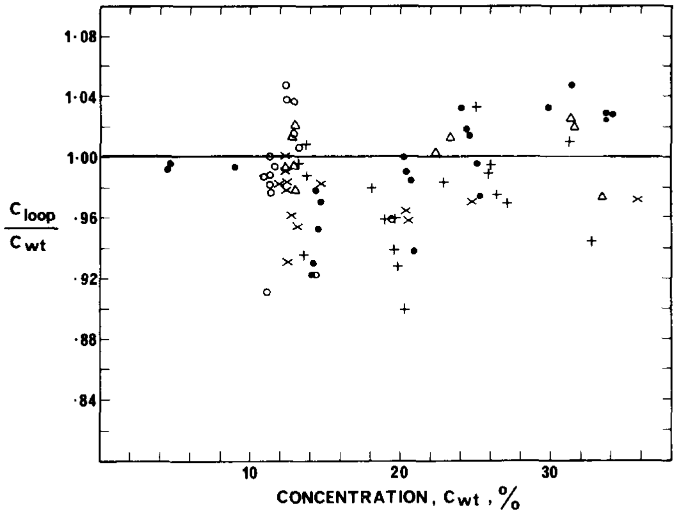

Figure 8. Comparison of concentration estimates by vertical loop and weigh tank.

discrepancies are not necessarily due to inaccuracies in the loop measuring system since errors in specific weight in the weigh tank system are optimistically estimated to be ±0.5%. Further, with the settling slurries, concentration variations with time occur, resulting in the weigh tank sample concentrations being different to the mean delivered concentration. This could explain why the greatest percentage deviations occurred for the most settling type sand (sand E).

On the basis of these results, slurry mass or volume flowrates can be accurately measured by combined use of the vertical loop and the venturi meter. For finer slurries, use of the water discharge-coefficient for the venturi should give an estimate of flow rate within the scatter of the experimental discharge-coefficients, which was in all cases less than 1% (for 150 μm and finer sand slurries). This scatter appears to be a result of limitations of the weigh tank measuring system so that it is estimated that the accuracy of flow rate measurements with the venturi would be limited only by the accuracy of the pressure and the specific weight measurements. For coarser slurries, use of the discharge coefficient as a function of concentration as in figures 4–6 would give a similar expected accuracy, although the Reynolds number dependence must be calibrated for the most settling slurries (slurries D and E). It is expected that for slurries coarser than those reported here, a greater dependence of discharge coefficient on Reynolds number will occur and this can only be determined by careful calibration.

ANALYSIS OF VENTURI FLOW

The venturi-discharge coefficients tended to show the greatest deviation from the water value for the most settling type of slurry (sand E), while for the finest particles the water value applied. It seems likely that the relative slip between the particles and the fluid contributes to this deviation. To determine the magnitude of slip effects necessary to explain these values a one-dimensional analysis of flow through the venturi has been carried out, where the relative velocities are represented by a slip ratio, S defined as the ratio of average solid velocity to average liquid velocity across the section.

By combining the momentum and continuity equations for slurry flow through a variable area pipe, the following non-dimensional relation is derived in the appendix:

$$-\frac{1}{\bar{p}}\frac{\mathrm{d}\bar{p}}{\mathrm{d}\eta} = D\left(2f - \frac{1}{\alpha}\frac{\mathrm{d}\alpha}{\mathrm{d}\eta}\right) + \frac{1}{S}\frac{\mathrm{d}S}{\mathrm{d}\eta}(1-q)(D-D_1), \qquad [5]$$

where $D = (\rho U^2)_m / p$;
$D_l = \rho_l U_l^2 / p$;
f = friction factor;
$\bar{p} = p/p_0$;
q = in-line solids concentration by volume;
U = velocity;
x = axial position;
$\alpha = A/A_0$;
$\eta = x/d$;
ρ = density;
$(\rho U^2)_m = q\rho_s U_s^2 + (1-q)\rho_l U_l^2$.
Subscripts l = liquid;
s = solid;
0 = reference condition.

This relation provides a simple illustration of the dependence of the pressure gradient on area changes, frictional losses and changes in the slip ratio. For a constant slip ratio the last term becomes zero.

Equation [5] has been solved numerically for the geometry of the venturi (figure 1) between the inlet and throat pressure tappings, and a discharge coefficient calculated by the definition [1]. If friction is neglected, and slip ratios are assumed to be unity (i.e. the mixture flows homogeneously) then the discharge coefficient will be equal to one. Deviations of this theoretical discharge-coefficient from unity will thus indicate the effect that various types of velocity ratios might have on the discharge coefficients.

Figure 9 shows the variation of this theoretical discharge-coefficient with solids concentration for the sand and ilmenite slurries tested, for various assumed slip functions. Curves A and B are for a constant velocity ratio, while for curve C, S increases linearly in the converging section from 0.8 to 1. It can be seen that a decrease in discharge coefficient may be the result of the velocity ratio approaching unity (as in curve C) as the slurries travel from the inlet to the venturi throat. That the assumed value of slip at the venturi inlet is realistic can be verified by the results of

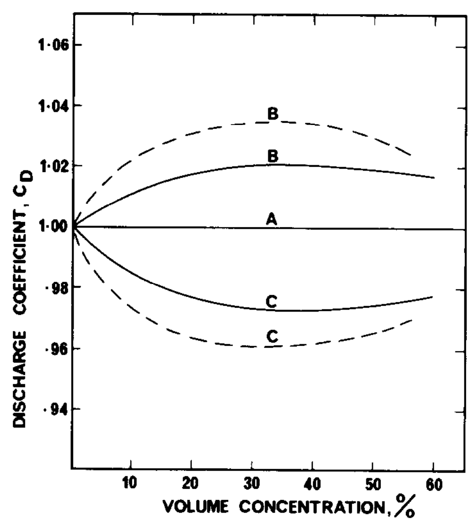

Figure 9. Theoretical discharge-coefficients. Reynolds number = 3×10^5.
$A: S = 1$
$B: S = 0.8$
$C: S = 0.8 + 0.2X/L$.
X/L = dimensionless distance along converging section.
—— sand;
--- ilmenite.

Newitt *et al.* (1962). They suggested an empirical relation for the mean slip velocity of the form

$$U_l - U_s = 1.26 - \frac{0.012(1-C)U_m}{w_0}, \qquad [6]$$

where U_m = total volume flow per unit area;
w_0 = free fall velocity of particles.

This correlation was based on results with a coarse gravel slurry and with a sand slurry very similar to sand E of this study, in a 25 mm pipe. Although the inlet diameter of the venturi in this study is 50 mm, the results of Newitt *et al.* (1962) should provide an approximation of the relative velocities. Equation [6] can be combined with definitions of the flow properties to give the velocity ratio, and for sand E in the inlet, velocity ratio was computed to be dependent mainly on velocity (and hence Reynolds number). For the range of Reynolds numbers in figure 8, the velocity ratio is estimated to increase with Reynolds number, from 0.6 to 0.85. Equation [6] was used to estimate velocity ratios at the venturi inlet and throat for use in [5], and the results in figure 9 have been re-computed using these new estimates, again assuming a linear variation of velocity ratio with axial location. These results have been plotted in figure 10 which shows that although inlet velocity ratios are dependent on flow velocity, the theoretical discharge-coefficient shows a lesser dependence. While the inlet values of velocity ratio estimated from [6] would be realistic, there is no reason for suggesting that calculated values at the throat should be representative because the flow in the throat would not be equivalent to a fully developed pipe flow. In order to explain the low discharge-coefficients at high concentration, velocity ratios at the throat higher than those estimated by [6], possibly greater than unity, would be necessary.

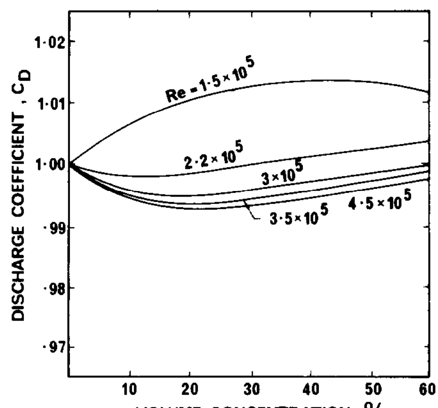

Figure 10. Theoretical discharge coefficients using slip ratios from Newitt *et al.* (1962).

VELOCITY RATIOS

There are two factors which cause velocity ratios to deviate from unity. The simplest to visualise is the local relative velocity between particles and the surrounding liquid. Obviously, if all particles are travelling slower than the surrounding fluid then the mean solids velocity will be below the mean liquid velocity, giving a velocity ratio less than one. The second factor is the effect of phase and velocity distributions, which even for purely axisymmetric flows can lead to overall relative velocity in the absence of local relative velocity. For stratified types of phase distributions which would occur for slurries containing sand *E* (see Newitt *et al.* (1962) for typical velocity and concentration profiles) the phase and velocity distributions contribute significantly to overall relative velocity.

Shook & Masliyah (1974) in a one-dimensional analysis of venturi flow considered changes in velocity ratio due to local slip. They allowed for additional particle drag forces due to the liquid acceleration, so that particle accelerations lagged behind fluid accelerations with consequent low velocity ratios at the venturi throat. This lead to high theoretical indicated discharge coefficients.

To investigate relative velocity aspects more fully, a visual study of flow through a perspex venturi with the dimensions on figure 1 was undertaken. The flow was photographed at the speed of 3500 frames per sec for sand E, showing the flow at inlet and throat, and in the diverging passage. Concentrations up to 20% by volume were photographed and individual particle trajectories were followed to allow estimates of the particle velocities. The flows were highly stratified at both inlet and throat so that particles were relatively densely packed in the bottom portion of the pipe. Estimated velocities of particles in this region may suffer in accuracy since it is possible that the films only show particles near the wall, but in the upper region of the pipe it was possible to follow particles away from the wall by choosing those which appeared to move with the main bulk of the flow. A typical result of these velocity measurements is given in figure 11. For the inlet velocities there appears to be significant scatter, with particle velocities being generally below the mixture velocity. At the throat however, the velocities appear more uniform and more closely approach the mixture velocity. This suggests that particle acceleration does not lag behind fluid acceleration, but rather that the particles accelerate faster than the fluid. This is borne out by the fact that the average value of the ratio of the individual particle velocities at inlet to throat for figure 11 is 2.93, whereas the area ratio (throat to inlet) is only 1.82. The result of this is an increase in velocity ratio at the throat which accounts for the low values of discharge coefficient at these higher concentrations. For lower concentrations, such large velocity increases were not evident since for 3.5% concentration, the particle velocity ratio was 1.93.

The relatively high increase in the particle velocities at the venturi throat is probably the result of a movement of the solids towards the centre of the pipe. This would also occur for the vertical-flow orientations and explain the fact that upward and downward flow discharge coefficients may tend to be below the water value. However, this tendency is also counter-balanced by the local slip effect as considered by Shook & Masliyah, possibly explaining the relatively high values of discharge coefficient for the coarsest slurry, sand E. For coarse slurries outside the limits of these experiments, careful calibration is recommended.

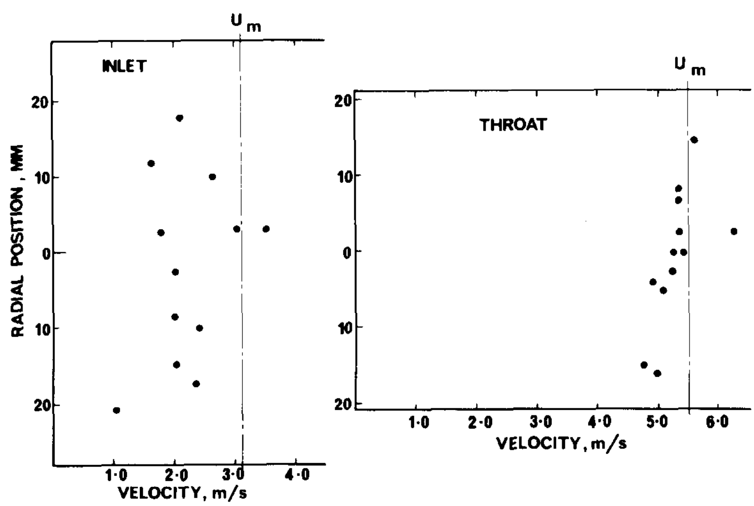

Figure 11. Particle velocities in venturi for slurry E at 20% concentration.

VENTURI ENERGY LOSSES

By causing a restriction to the main flow, venturi meters cause a pressure loss and values of losses for single phase fluids are well documented in BS 1042 (1964). For the venturi discussed in this paper, the energy loss for water was found experimentally to be 17% of the velocity head, or alternatively 8% of the inlet to throat pressure difference, in agreement with Standards. Graf (1967) suggested that with slurries, the additional pressure loss across the venturi due to the presence of the solids is directly proportional to solids concentration, independent of velocity. To investigate this fact, pressure profiles were obtained for sand E at 5% and 18% concentration and values of the energy loss determined. For both of these two conditions it was found that the additional energy loss was almost directly proportional to velocity at each concentration so that Graf's postulate was not verified by these tests. It is possible however, that after suitable calibration, the energy loss measurements may be considered simultaneous to inlet to throat measurements, to yield both flowrate and concentration, but this would require more extensive energy loss measurements than those undertaken here.

CONCLUSIONS

The experimental study has indicated that for sand slurries with a median diameter below about 150 μm, a venturi meter of similar geometry to that in figure 1 can be used for metering flow rates, with a discharge coefficient equivalent to the water value. No calibration would be necessary, and accuracy should be equivalent to that for single phase fluids. For concentration measurements which are required for the flow rate measurements, the vertical loop section appeared satisfactory for all size ranges by giving estimates of mixture specific gravity within the range of uncertainty of the calibrating procedure.

For coarser slurries, the discharge coefficients of figures 4–6 can be used as a guide, but generally the venturi meter could be calibrated accurately for a particular application. The similarity of the discharge coefficients for the ilmenite slurry and for the sand C slurry indicates that the particle settling velocity can be used as a guide to the deviation of the discharge coefficient from the water values, although this aspect needs further study.

The cause of deviations of discharge coefficient from water values has been shown to be the change in relative velocity between the particles and the water, most likely resulting from a movement of particles towards the centre of the flow. This does not limit the applicability of venturi meters, but simply suggests the need for calibration with coarse slurries.

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REFERENCES

BEHREND, G. M. 1973 Where we stand in slurry flow measurement. The Canadian Min. and Met. Bulletin, CIM. September, 70–75.

BRITISH STANDARDS INSTITUTION 1964 Methods for the measurement of fluid flow in pipes. BS 1042, Part 1.

BROOK, N. 1962 Flow measurement of solid–liquid mixtures using venturi and other meters. *Proc. Instn Mech. Engrs* **176**, 127–140.

EINSTEIN, H. A. & Graf, W. A. 1966 Loop system for measuring sand–water mixtures. *Proc. ASCE, J. Hyd. Div.* **92**, 1–12.

GRAF, W. A. 1967 A modified venturi meter for measuring two-phase flow. *IAHR, J. Hyd. Res.* **5**, 161–187.

NEWITT, D. M., RICHARDSON, J. F. & SHOOK, C. A. 1962 Hydraulic conveying of solids in horizontal pipes, Part II: Distribution of particles and slip velocities. *Proceedings of the Symposium on the Interaction between Fluids and Particles*, pp. 87–100. Institution of Chemical Engineers, London.

PEIRCE, J. W. 1962 Mass flow measurement of mining slurries. *Trans. Soc. Mining Engrs* March 34–37.

SHOOK, C. A. & MASLIYAH, J. H. 1974 Flow of a slurry through a venturi meter. *Can. J. Chem. Engng* **52**, 228–233.

VAN DER VEEN, R. 1972 The development of an accurate concentrationmeter for two phase flow. Second Int. Conf. on Hydraulic Transport of Solids, Univ. of Warwick, BHRA, Cranfield, Bedford, England.

WEISMAN, N. B. & GRAF, W. H. 1968 Measuring solid–liquid mixture in closed conduits, *Proc. ASCE, J. Waterways & Harbours Div.* **94**, 453–464.

APPENDIX

The basic equations for flow of a two phase incompressible mixture through a variable area duct will be derived. The analysis is a one-dimensional approach which allows the velocities of the phases to differ. The forces acting on a control volume with steady mass-flow are as indicated in figure A1. Equating the net force to the rate of change of momentum of the two phases gives:

$$\tau_w \cdot \pi d \cdot \mathrm{d}x + \left(p + \frac{\mathrm{d}p}{\mathrm{d}x}\mathrm{d}x\right)\left(A + \frac{\mathrm{d}A}{\mathrm{d}x}\mathrm{d}x\right) - pA - \left(p + \frac{\mathrm{d}p}{\mathrm{d}x}\frac{\mathrm{d}x}{2}\right)\frac{\mathrm{d}A}{\mathrm{d}x} \cdot \mathrm{d}x + q\rho_s A U_s \frac{\mathrm{d}U_s}{\mathrm{d}x} \cdot \mathrm{d}x + (1-q) - \rho_1 A U_1 \frac{\mathrm{d}U_1}{\mathrm{d}x} \cdot \mathrm{d}x = 0 \quad \text{[A1]}$$

where A = cross-sectional flow area;
q = average in-line solids concentration;
d = diameter;
τ_w = wall shear stress;
p = static pressure.

Equation [A1] simplifies to

$$-\frac{\mathrm{d}p}{\mathrm{d}x} = \frac{4\tau_w}{d} + q\rho_s U_s \frac{\mathrm{d}U_s}{\mathrm{d}x} + (1-q)\rho_1 U_1 \frac{\mathrm{d}U_1}{\mathrm{d}x}. \quad \text{[A2]}$$

From continuity for each phase

$$\frac{\mathrm{d}}{\mathrm{d}x}(Aq\rho_s U_s) = 0 \quad \text{[A3]}$$

and

$$\frac{\mathrm{d}}{\mathrm{d}x}(A(1-q)\rho_1 U_1) = 0 \quad \text{[A4]}$$

which in turn gives

$$-\frac{\mathrm{d}U_s}{\mathrm{d}x} = \frac{U_s}{A}\frac{\mathrm{d}A}{\mathrm{d}x} + \frac{U_s}{q}\frac{\mathrm{d}q}{\mathrm{d}x} \quad \text{[A5]}$$

and

$$-\frac{\mathrm{d}U_1}{\mathrm{d}x} = \frac{U_1}{A}\frac{\mathrm{d}A}{\mathrm{d}x} + \frac{U_1}{(1-q)}\frac{\mathrm{d}q}{\mathrm{d}x} \quad \text{[A6]}$$

and which can be combined to eliminate the term in $\mathrm{d}q/\mathrm{d}x$ as

$$-\frac{\mathrm{d}U_1}{\mathrm{d}x} = \frac{U_1}{A}\frac{\mathrm{d}A}{\mathrm{d}x} + \frac{U_1 q}{(1-q)}\left\{\frac{1}{U_s}\frac{\mathrm{d}U_s}{\mathrm{d}x} + \frac{1}{A}\frac{\mathrm{d}A}{\mathrm{d}x}\right\}. \quad \text{[A7]}$$

If the slip ratio, S, is now introduced as the ratio of solids velocity to liquid velocity, it can be shown that

$$\frac{\mathrm{d}U_s}{\mathrm{d}x} = S\frac{\mathrm{d}U_1}{\mathrm{d}x} + U_1\frac{\mathrm{d}S}{\mathrm{d}x} \quad \text{[A8]}$$

and

$$\frac{\mathrm{d}U_1}{\mathrm{d}x} = \frac{1}{S}\frac{\mathrm{d}U_s}{\mathrm{d}x} - \frac{U_1}{S}\frac{\mathrm{d}S}{\mathrm{d}x}. \quad \text{[A9]}$$

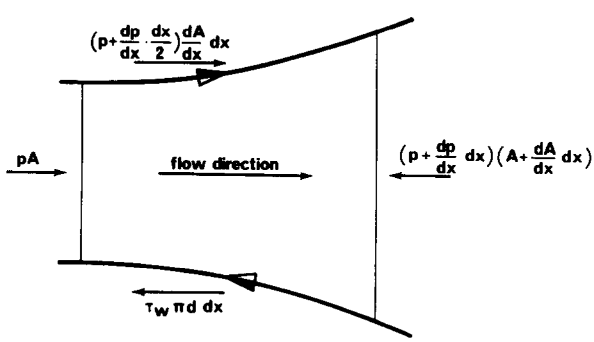

Figure A1. Control volume for variable area pipe flow.

This allows [A7] to be written in the alternative forms

$$\frac{dU_s}{dx} = (1-q)U_1\frac{dS}{dx} - \frac{U_1 S}{A}\frac{dA}{dx} \qquad [A10]$$

and

$$\frac{dU_1}{dx} = -\frac{U_1}{A}\frac{dA}{dx} - \frac{qU_1}{S}\frac{dS}{dx}. \qquad [A11]$$

Equations [A10] and [A11] can now be substituted into [A2] which becomes

$$-\frac{dp}{dx} = \frac{4\tau_w}{d} - \frac{1}{A}\frac{dA}{dx}(\rho U^2)_m + \frac{1}{S}\frac{dS}{dx}q(1-q)(\rho_s U_s^2 - \rho_1 U_1^2) \qquad [A12]$$

where

$$(\rho U^2)_m = q\rho_s U_s^2 + (1-q)\rho_1 U_1^2. \qquad [A13]$$

If a friction factor is now defined by the expression

$$\tau_w = f \cdot \tfrac{1}{2}(\rho U^2)_m \qquad [A14]$$

then [A12] becomes

$$-\frac{dp}{dx} = \left(\frac{2f}{d} - \frac{1}{A}\frac{dA}{dx}\right)(\rho U^2)_m + \frac{1}{S}\frac{dS}{dx}q(1-q)(\rho_s U_s^2 - \rho_1 U_1^2). \qquad [A15]$$

This relation can be non-dimensionalised to give

$$-\frac{1}{\bar{p}}\frac{d\bar{p}}{d\eta} = D\left(2f - \frac{1}{\alpha}\frac{d\alpha}{d\eta}\right) + \frac{1}{S}\frac{dS}{d\eta}(1-q)(D - D_1) \qquad [A16]$$

where $\alpha = A/A_0$;
$\bar{p} = p/p_0$;
p_0 = reference pressure;
$\eta = x/d$;
$D = (\rho U^2)_m/p$;
$D_1 = \rho_1 U_1^2/p$.

It should be noted that in the appendix, all quantities referred to have been cross-sectional average values because of the one-dimensional approach. The effects of concentration and velocity distributions are included in the slip ratios.